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*Editorial*

# Mathematics and Applications

**Paolo Mercorelli [1,*] and Francisco Chiclana [2]**

[1] Institute for Production Technology and Systems (IPTS), Leuphana Universität Lüneburg, 21335 Lüneburg, Germany
[2] School of Computer Science and Informatics, De Montfort University, The Gateway, Leicester LE1 9BH, UK; chiclana@dmu.ac.uk
* Correspondence: paolo.mercorelli@leuphana.de

## 1. Motivation and Technical Context

Mathematics has long served as a universal language for understanding, modelling, and shaping the world around us. Its strength lies not only in its internal coherence and abstraction, but also in its remarkable ability to describe, analyse, and solve problems arising across a wide spectrum of disciplines. From the natural sciences to engineering, and from finance to sociology, mathematical methods continue to provide the foundation for innovation, discovery, and technological advancement.

This Special Issue, "Mathematics and Its Applications", reflects this broad and dynamic landscape. It also celebrates the 1st International Electronic Conference on Mathematics and Applications, held from 1 to 15 May 2023, which brought together researchers working at the intersection of theory and application. The contributions collected here highlight the diversity of modern mathematics, ranging from analytical and algebraic developments to computational techniques and interdisciplinary applications.

A central aspect of this Special Issue is the role of applications not only as an endpoint of mathematical research, but also as a means of deepening theoretical understanding. Real-world problems often challenge existing models, reveal limitations, and inspire the development of new mathematical concepts and methods. In this sense, applications act as a powerful driver of theoretical progress, providing validation, refinement, and new perspectives on established approaches. Moreover, the increasing relevance of algorithms and computational methods underscores the significant social and economic impact of applied mathematics. From data analysis and decision-making systems to optimisation in industry and finance, mathematical algorithms play a crucial role in shaping modern societies. Their influence extends to areas such as resource allocation, technological innovation, and policy development, highlighting the responsibility and importance of rigorous, transparent, and efficient mathematical modelling.

The scope of the issue encompasses a wide array of topics, including engineering mathematics, computer science, dynamical systems, financial mathematics, mathematical physics, algebra and geometry, probability and statistics, mathematical biology, network science, fuzzy set theory, differential equations, and computational mathematics. Despite their diversity, these areas share a common goal: to address complex and emerging problems through rigorous mathematical frameworks and innovative approaches.

By gathering contributions from both conference participants and the broader scientific community, this Special Issue aims to provide a comprehensive overview of current trends and challenges in applied mathematics. It seeks not only to disseminate recent advances but also to foster cross-disciplinary dialogue, encouraging new connections between fields and inspiring future research directions.

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## 2. Contributions of the Special Issue

Contribution 1 analyses adaptive control approaches for nonlinear systems affected by uncertainties, nonlinear behaviours, and component faults. The discussion covers different classes of nonlinear systems, including stochastic and switched systems, emphasising the difficulties involved in their modelling and control. Particular attention is given to nonlinear effects such as input saturation, dead zones, hysteresis, and failures in actuators or sensors, all of which can significantly degrade system performance. The work also examines the use of fuzzy logic and neural networks as approximation tools capable of handling uncertain dynamics. Finally, the review summarises recent progress in fault-tolerant adaptive control and outlines future research directions focused on real-time learning and improved fault detection.

The study in contribution 2 focuses on improving the efficiency of large language models during inference, especially in long-context tasks where memory usage becomes a major limitation. The proposed method introduces an entropy-based strategy for allocating the key–value cache inside Transformer layers. Layers with more distributed attention patterns are assigned larger cache budgets, while layers with concentrated attention receive fewer resources. Instead of selecting tokens independently for each attention head, the approach identifies a shared set of relevant tokens for every layer, preserving the structure of multi-head attention while reducing computational cost. Experimental evaluations show that the strategy decreases memory consumption and accelerates decoding without reducing generation quality.

Contribution 3 revisits Equilibrium Propagation, a biologically inspired alternative to backpropagation for recurrent neural networks. Although the Directed Equilibrium Propagation model removes the need for perfectly symmetric feedback connections, it still suffers from instability and convergence problems. To address these issues, the authors introduce neuronal leakage into the framework and derive new conditions that guarantee stable dynamics. They also propose a local learning rule connected to the optimisation objective, improving the reliability of learning in small neural networks. The results contribute to making energy-based learning methods both more biologically plausible and more theoretically grounded.

In contribution 4, a probabilistic framework for Parametrized Relativistic Quantum Theory (PRQT) has previously been established for multicomponent eigenvector systems and many-body formulations in flat spacetime configurations. This work extends the existing multicomponent and many-body PRQT formalism to the more general context of curved spacetime, enabling the theoretical framework to incorporate gravitational effects and non-Euclidean geometrical structures.

The study in contribution 5 introduces a DGC-t-MSV model that incorporates both dynamic correlation structures and Granger causality within a multivariate stochastic volatility framework in order to investigate volatility spillover mechanisms between China's carbon market and traditional manufacturing sectors. Using daily closing price data collected between January 2022 and November 2024, the analysis examines the interactions between the carbon market and multiple industrial sectors from the perspective of industrial heterogeneity. The empirical results indicate the presence of significant unidirectional volatility spillovers from the carbon market toward highly carbon-intensive industries, including steel, chemical production, shipbuilding, and automobile manufacturing, identifying the carbon market as the primary source of volatility transmission. In contrast, sectors such as forest products, textiles, construction engineering, and machinery manufacturing exhibit bidirectional spillover dynamics, although in these cases the carbon market mainly behaves as a receiver of volatility shocks. The study further reveals that the intensity of dynamic correlations between the carbon market and manufacturing industries is posi-

tively associated with sectoral carbon emissions. Stronger correlations are observed for industries characterised by high emission levels, particularly steel, chemicals, machinery manufacturing, construction engineering, and automobile production, whereas weaker relationships emerge in the textile and forest product sectors. Moreover, the analysis highlights that the carbon market displays substantially higher volatility than the traditional manufacturing industries considered in the study. Overall, the work provides a novel sector-specific perspective on volatility transmission mechanisms and offers relevant policy implications for the coordinated development of carbon-trading systems and industrial manufacturing activities.

Contribution 6 investigates a class of self-repelling diffusion processes driven by Lévy noise, extending the analysis of stochastic systems with memory-dependent dynamics and jump discontinuities. By employing a coupling-based probabilistic approach, the authors derive the associated Bismut-type formula and establish a corresponding Harnack inequality. These results provide important analytical tools for studying the regularity, stability, and transition properties of the underlying stochastic process, particularly in the context of non-local diffusion dynamics generated by Lévy processes.

Contribution 7 revisits the group classification problem for a class of nonlinear heat and diffusion equations characterised by variable diffusion behaviour. The study highlights the importance of group classification methods as tools for identifying the functional structures and parameters that maximise the symmetry properties of partial differential equations. The authors develop a complete classification framework by deriving the determining equations associated with the infinitesimal generators of the admitted Lie symmetry groups and systematically solving the resulting constraints for the admissible diffusion functions. The analysis reproduces the classical results originally obtained by Ovsyannikov and Bluman while providing a clearer and more detailed derivation of the mathematical procedure involved. The classification identifies multiple distinct cases, each associated with a specific diffusion structure and a corresponding Lie symmetry algebra of a different dimension. The work therefore offers a comprehensive symmetry-based characterisation of nonlinear diffusion models and clarifies how the choice of diffusion behaviour influences the algebraic and geometric properties of the equation.

Contribution 8 investigates the consistency between approximate non-equilibrium Green's function methods and approximate solutions of the Dyson equation within the quantum theory of many-particle systems. The analysis focuses on their compatibility with a modified form of the continuity equation, which differs from the standard conservation framework typically adopted in conventional electrodynamics. The study shows that this generalised continuity relation permits electromagnetic coupling within the framework of extended Aharonov–Bohm electrodynamics, while remaining incompatible with standard Maxwell electrodynamics. Particular attention is devoted to molecular junctions modelled numerically through density functional theory, where the authors examine the implications of alternative definitions of local current density. In this context, the paper discusses the reformulation proposed by Wang and collaborators, demonstrating that the modified current-density description is also compatible with the extended Aharonov–Bohm theoretical framework.

Contribution 9 investigates the development of a non-self-referential version of the Gram–Schmidt process for transforming a set of linearly independent vectors into an orthogonal basis. The classical Gram–Schmidt procedure is widely used in algebra and applied mathematics for constructing orthonormal bases, QR decompositions, and unitary matrices, with important applications in areas such as communications, machine learning, and feature extraction. However, standard formulations rely on recursive self-referential computations, whereas existing non-self-referential approaches based on Gram determi-

nants suffer from exponential computational complexity. To address this limitation, the authors derive a new non-self-referential formulation by exploiting the recursive structure underlying the standard Gram–Schmidt process. The resulting method provides both individual and collective representations of the orthogonalised vectors in a compact matrix form while significantly reducing computational complexity from exponential to polynomial order. In addition to its improved computational efficiency, the proposed framework offers a more elegant and structured mathematical representation. The paper further evaluates the proposed method through a classification problem involving real-world datasets. Experimental results indicate that the approach achieves more convincing and competitive performance compared with several commonly adopted methods, highlighting its practical applicability in data analysis and machine learning contexts.

Contribution 10 proposes a novel entropy-based framework for solving constrained linear regression problems. Instead of relying on the conventional minimisation of quadratic error, the method formulates the regression task as an entropy minimisation problem based on the Fermi–Dirac entropy, while incorporating the observed data as constraints within the optimisation procedure. A key aspect of the proposed approach is its ability not only to estimate the regression coefficients but also to simultaneously infer measurement errors. The framework assumes only limited prior information about these errors, namely the expected interval within which they are likely to lie, similarly to the assumptions imposed on the unknown regression parameters. This results in a flexible estimation strategy that avoids stronger probabilistic assumptions commonly adopted in classical regression models. The authors compare the performance of the proposed methodology with solutions obtained through disciplined convex optimisation techniques and further analyse issues related to the consistency and reliability of the estimates. Several numerical examples are presented to demonstrate the effectiveness and practical applicability of the entropy-based regression framework.

Contribution 11 presents a detailed analysis of several widely used normalised similarity and distance measures for Boolean models. The study focuses on three major classes of measures, namely generalised Kulczynski, generalised Jaccard, and generalised Consonni–Todeschini indices, extending their formulations and examining their mathematical relationships within a unified framework. The authors establish a theoretical ordering both among similarity measures belonging to the same class and across different classes, providing a systematic comparison of their structural properties and relative behaviours. In addition, the work investigates the metric characteristics of these measures, with particular attention to the conditions under which they satisfy or violate the triangle inequality property. The analysis includes both positive and negative results regarding metric validity, clarifying which similarity and distance measures can be consistently interpreted within a proper metric-space framework. Overall, the paper contributes to a deeper theoretical understanding of Boolean similarity measures and their applicability in classification, pattern recognition, and data analysis problems.

Contribution 12 addresses the problem of counting independent sets in graphs, a fundamental challenge in combinatorial mathematics with important applications in computer science, physics, and chemistry. In the context of chemical graph theory, the number of independent sets corresponds to the Merrifield–Simmons index, a topological descriptor strongly related to physical properties such as the boiling point of chemical compounds. The authors propose a new algorithm for counting independent sets in grid-like graph structures. The method is based on a branch-and-bound strategy and is specifically applied to square grids organised into rows and columns. The approach incorporates the classical vertex reduction rule as the main mechanism for recursively partitioning the graph into smaller subgraphs. Through an iterative decomposition process, the original grid is pro-

gressively reduced until outerplanar configurations are obtained or until specific structural conditions on internal faces are no longer satisfied. These reduced structures serve as the terminal cases of the computational procedure. The proposed branch-and-bound framework significantly improves computational efficiency compared with traditional techniques such as the transfer matrix method. Moreover, the algorithm is not restricted to regular grid structures and can be extended to more general mesh graphs, making the methodology applicable to a broader class of combinatorial and topological problems.

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